The effect of the streamwise gradient of the normal stress on the computed results is small and apparently may be neglected.

References

[1] Bushnell, D. M. and Morris, D. J., "Shear-Stress, Eddy-Viscosity and Mixing Length Distributions in Hypersonic Turbulent Boundary Layers," TMX-2310, Aug. 1971, NASA.

[2] Horstman, C. C. and Owen, F. K., "Turbulent Properties of a Compressible Boundary Layer," *AIAA Journal*, Vol. 10, No. 11, Nov. 1972, pp. 1418–1424.

[3] Sturek, W. B., "Calculations of Turbulent Shear Stress in Supersonic Turbulent Boundary-Layer Zero and Adverse Pressure Gradient Flow," AIAA Paper 73-166, Washington, D.C., 1973.

[4] Johnson, D. A. and Rose, W. C., "Measurement of Turbulent Transport Properties in a Supersonic Boundary-Layer Flow Using Laser Velocimeter and Hot Wire Anemometer Techniques," AIAA Paper 73-1045, Seattle, Wash., 1973.

[5] Rose, W. C. and Johnson, D. A., "A Study of Shock Wave Turbulent Boundary-Layer Interaction Using Laser Velocimeter and Hot Wire Anemometer Techniques," AIAA Paper 74-95, Washington, D.C., 1974.

[6] Sun, C. C. and Childs, M. E., "A Modified Wall-Wake Velocity Profile for Turbulent Compressible Boundary Layers," *Journal of Aircraft*, Vol. 10, No. 6, June 1973, pp. 381–383.

[7] Maise, G. and McDonald, H., "Mixing Length and Kinematic Eddy Viscosity in a Compressible Boundary Layer," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 73–80.

Effects of Discontinuities in Cylindrical Tubes on a Transmitted Pulse

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Nomenclature

L = length of the area discontinuity
λ = pulse wavelength
σ_T = transmitted stress measured at position X_2
σ_I = incident stress measured at position X_1
A_1 = larger cross-sectional area
A_2 = reduced cross-sectional area

Introduction

THE literature on the effects of discontinuities in cylindrical shells is rather limited. Several investigators[1,2] have considered the effects of shells with discontinuous areas on acoustic transmission. The free vibration of a thin cylindrical shell with a discontinuity was studied by Warburton and Al-Mojafi.[3] The effects of discontinuity in bars was studied by Ripperger and Abramson[4] and Kenner and Goldsmith,[5] where the ratio of the transmitted to incident stress was related to the geometrical properties of the bar before and after the discontinuity. In a recent paper, Mortimer, Rose, and Blum[6] obtain a similar relationship between the transmitted to incident stress and the geometrical properties before and after the discontinuity for shells. The analytical results were obtained by numerically solving a general bending shell theory and the results were experimentally verified. This approach was further developed by Rose and Mortimer[7] to obtain a relationship between the transmitted to incident stress in cylindrical shells consisting of a double area discontinuity. The experimental and analytical results obtained in Ref. 7 were in good agreement. However the discontinuity and the pulse duration were rather long. For short discontinuity ($L < \lambda/2$), the effect of the discontinuity on the transmitted pulse is more apparent and the pulse length is more significant. The effects of the pulse length on the propagating pulse in cylindrical shells was recently studied by Mortimer and Blum,[8] and it was shown to be a major parameter in determining the shell response to impact loading.

In this Note the effects of relatively long and short discontinuities are studied. For a relatively long discontinuity ($L \geqq \lambda/2$) a good agreement between experimental results, using ultrasonic technique, and analytical results using the equation developed in Ref. 7, is observed. However, for a relatively short discontinuity ($L < \lambda/2$) due to reflections off the discontinuities, superposition of the wave is now present, which makes the transmitted wave differ in both magnitude and shape from the incident wave. The equation developed in Ref. 7 is no longer valid. Thus, a computer program MCDU-26[6] was used to solve the set of governing equations. Because the frequency of the input pulse is now a function of the magnitude of the transmitted pulse, one computer run was needed to solve the problem for each input frequency. However, if a pulse whose spectral representation consists of a broad range of frequencies is used as input in MCDU 26, and the Fourier series of the transmitted pulse is taken, we now have not only the magnitude of the transmitted pulse of the frequency of the input, but also the magnitudes of the transmitted pulses of all the frequencies that represent the input pulse in the Fourier series. Design curves of various analytical models can be drawn relating frequency to the magnitudes of the incident and transmitted pulses. To check the accuracy of this technique, full sine continuous waves of various frequencies were input pulses to MCDU 26, and the magnitudes of incident and transmitted pulses from these runs showed good agreement to those generated from the design curves.

Investigation and Procedures

The study presented in this Note can be grouped into three investigations. Several experimental and analytical models containing area discontinuities were used. The purpose of the study was to investigate the effects of a relatively long and short discontinuity on the magnitude of the transmitted pulse in a cylindrical tube. This pulse can be related to a stress pulse transmitted through the discontinuity and for clarity in comparison will be referred to hereafter as the magnitude of the transmitted stress.

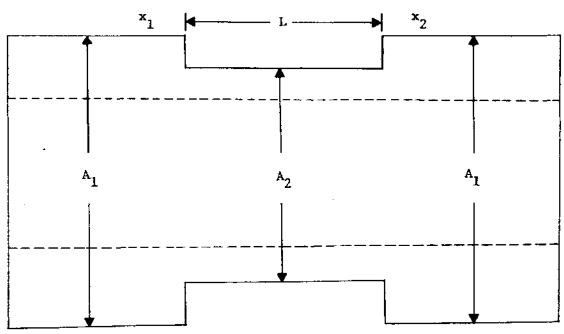

Fig. 1 Schematic of test model.

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In addition, the effects of higher frequency and short discontinuity on the transmitted pulse were studied. The Fourier series representation was used to describe the pulse before and after the discontinuity and design curves relating frequency, area ratio, and discontinuity length are developed. The final investigation in this study was to develop a technique in which one computer run will be sufficient to obtain the magnitude of the transmitted pulse generated from a broad range of frequencies. This will save numerous computer runs.

The first investigation was to study the effects of a relatively long discontinuity ($L \geqq \lambda/2$) on the magnitude of the transmitted stress. For this study several steel cylindrical tubes with various geometrical properties were selected. A typical schematic of a model is shown in Fig. 1. The theoretical results were obtained by measuring an ultrasonic signal transmitted through the tube. The signal is measured at the locations 0.5 in. (127 mm) before and after the discontinuity, which will determine the input and transmitted pulse respectively. The input pulse was 1 MHz full sine with 1 μsec period for all models; and in addition for Model No. 1 a 4 MHz half $\sin^2$ with 0.25 μsec period input pulse was used. The analytical results were obtained utilizing the equation developed by Rose and Mortimer[7] and is repeated here:

$$\sigma_T/\sigma_I = 4(A_1/A_2)/(1+A_1/A_2)^2 \tag{1}$$

The geometrical parameters and the results of this investigation are summarized in Table 1.

Table 1 Geometrical properties and stress ratios σ_T/σ_I

Model no.	Input function[a]	A_1/A_2	L	σ_T/σ_I, exp	σ_T/σ_I, Eq. (1)
1	A	1.5	2.75	0.955	0.96
	B			0.957	
2	A	1.5	1.75	0.955	0.96
3	A	4.0	2.75	0.625	0.64
4	A	1.5	2λ	0.968	0.96
5	A	1.5	$\lambda/2$	0.97	0.96

[a] Input function: A = 1 MHz; full-sine τ = 1.0 μsec. B = 4 MHz; half-$\sin^2$ τ = 0.25 μsec.

The second investigation consisted of studying the effects of a relatively short discontinuity ($L < \lambda/2$) and the effects of frequency on the magnitude of the transmitted stress. For this study we utilized the computer program MCDU 26[6] to obtain the numerical solution to the governing equation of motion for a cylindrical shell containing an area discontinuity. The input and transmitted pulse in a time domain were represented in a frequency domain by a Fourier series. From these representations the effect of the frequency on the magnitude of the transmitted pulse can be observed. In addition, design curves relating the magnitude ratio of input and transmitted stress to the frequency and geometrical properties of the tube are developed. The input pulse for this study for all models was 4 MHz half $\sin^2$ with 0.25 μsec period. The results of this investigation are summarized in Table 2. Design curves were developed for two discontinuity lengths of $L = \lambda/4$ (L = 0.013 in.) and $L = \lambda/10$ (L = 0.005 in.) and are shown in Fig. 2.

The third investigation in our study was to check the analytical technique of determining the effect of frequency on the magnitude of the transmitted pulse in a cylindrical shell of a given geometry. This was accomplished by simulating one of the geometries used to generate a design curve, and to run this geometry in MCDU 26 twice, using a different frequency pulse as input each time. The model having $A_1/A_2 = 4$ and L = 0.005 in. (This length corresponds to $\lambda/10$ for input pulse of 4 MHz half $\sin^2$ and 0.25 μsec period.) was selected for this investigation. Two input pulses were used, the first is a continuous 1 MHz full sine pulse and the second is 4 MHz full sine pulse. The magnitude of the pulse before and after the discontinuity is obtained and the stress ratio σ_T/σ_I is determined. Then we enter with these values σ_T/σ_I at the respective frequencies into the design curves in Fig. 2. The location of the points on the design curves indicate a good agreement between the two methods. The results of this investigation is summarized and compared with previous results in Table 3.

Table 3 Comparison of stress ratios

Input Frequency MHz	A_1/A_2	L, in.	σ_T/σ_I Full sine pulse	σ_T/σ_I Fourier series	% Error
1	4.0	0.005	0.977	0.928	5.28
4			0.739	0.713	3.64

Results and Conclusions

The results obtained from the first investigation (Table 1) show that for a relatively long discontinuity ($L \geqq \lambda/2$) a very good agreement between the experimental and analytical results as predicted by Eq. (1) is observed. The percent error varied from 0.52% for $A_1/A_2 = 1.5$ to 2.4% for $A_1/A_2 = 4.0$. In addition, we observed that increasing the frequency had very little effect on the magnitude of the transmitted pulse as shown in Table 1.

In the second investigation the effects of a relatively short discontinuity ($L < \lambda/2$) and frequency change are more apparent. Due to the reflections from the discontinuity, especially for higher frequency pulses, the magnitude of the transmitted stress is reduced considerably. The analytically predicted value obtained from Eq. (1) is no longer valid, as shown in Table 2. Observation of these results shows that as the frequency increases, the ratio of the transmitted to incident stress is decreasing, thus

Table 2 Geometrical properties and stress ratios σ_T/σ_I[a]

Model No.	A_1/A_2	L	σ_T/σ_I, Fourier series, Frequency, MHz: 1	2	3	4	5	6	7	σ_T/σ_I Eq. (1)
1A	1.5	$\lambda/4$	0.980	0.970	0.954	0.935	0.918	0.911	0.954	0.96
1B		$\lambda/10$	0.996	0.994	0.990	0.983	0.974	0.96	0.930	
2A	4.0	$\lambda/4$	0.770	0.710	0.610	0.470	0.320	0.411	1.070	0.64
2B		$\lambda/10$	0.928	0.886	0.815	0.713	0.582	0.436	0.463	
3A	6.0	$\lambda/4$	0.637	0.577	0.474	0.324	0.153	0.339	1.080	0.49
3B		$\lambda/10$	0.860	0.785	0.705	0.580	0.420	0.240	...	

[a] Input function: 4 MHz; half-$\sin^2$ τ = 0.25 μsec.

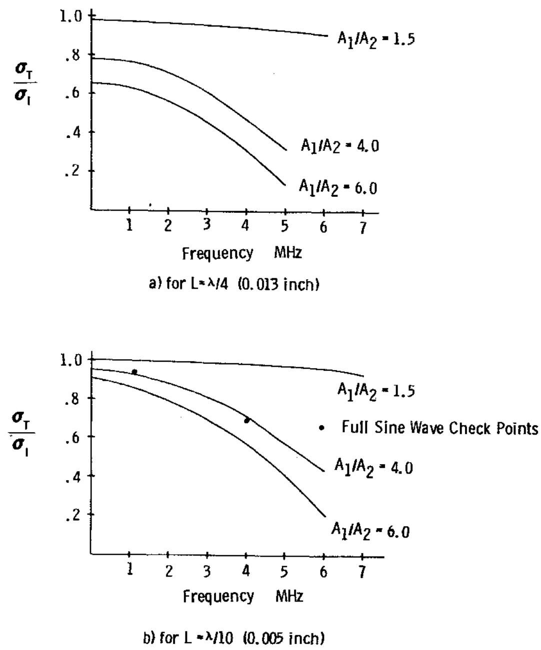

Fig. 2 Design curves—stress ratio vs frequency.

more of the input energy is transformed into the reflected modes of the propagating pulse through the discontinuity. Figure 2 presents the design curves developed from the results in Table 2. These design curves can be used as a tool to determine the loss in magnitude of the transmitted pulse for a given A_1/A_2 ratio and frequency value. These curves can also be used in the nondestructive testing of determining flaw sizes and voids as was suggested in Ref. 7.

The results of the third investigation verify that one computer run can be used to predict the effect of frequency on the stress ratio obtained in the second investigation with a big saving in computer runs. The results summarized in Table 3 show that good agreement is achieved between the two methods. The percent error varies from 3.64% to 5.28%, as indicated.

References

[1] Karl, F. C., "The Analogous Acoustical Impedance for Discontinuities and Constrictions of Circular Cross Section," *Journal of the Acoustical Society of America*, Vol. 25, No. 2, March 1953, pp. 327–334.

[2] Miles, J., "The Reflection of Sound due to a Change in Cross Section of a Circular Tube," *Journal of the Acoustical Society of America*, Vol. 16, No. 1, July 1944, pp. 14–19.

[3] Warburton, G. G. and Al-Mojafi, A. M. J., "Free Vibration of Thin Cylindrical Shells with a Discontinuity in the Thickness," *Journal of Sound and Vibrations*, Vol. 9, May 1969, pp. 373–382.

[4] Ripperger, E. A. and Abramson, H. N., "Reflection and Transmission of Elastic Pulses in a Bar of a Discontinuity in Cross Section," *Proceedings, Midwestern Conference on Solid Mechanics*, 3rd ed., 1957, pp. 135–145.

[5] Kenner, V. H. and Goldsmith, W., "One-Dimensional Wave Propagation Through a Short Discontinuity," *Journal of the Acoustical Society of America*, Vol. 45, No. 1, Aug. 1968, pp. 115–118.

[6] Mortimer, R. W., Rose, J. L., and Blum, A., "Longitudinal Impact of Cylindrical Shells with Discontinuous Cross-Sectional Area," *Journal of Applied Mechanics*, Vol. 39, No. 4, Dec. 1972, pp. 1005–1011.

[7] Rose, J. L. and Mortimer, R. W., "Elastic Wave Analysis in Nondestructive Testing," *Journal of Materials Evaluation*, Vol. XXXI, No. 3, March 1973, pp. 33–47.

[8] Mortimer, R. W. and Blum, A., "The Effect of Pulse Duration on the Transient Response of Cylindrical Shells Subjected to Axial Impact," *Journal of Applied Mechanics*, Vol. 41, March 1974, pp. 312–313.

MHD Augmented Shock Tunnel Experiments with Unseeded, High Density Air Flows

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Introduction

THE experimental study[1] referenced by Pate et al.[2] provides important and previously unpublished data which supports the feasibility of the high density, MHD augmented wind-tunnel concept. The results of this initial feasibility study are of particular interest in that they provide the only known available MHD augmented shock tunnel data obtained using unseeded air flows. Such operation more closely simulates real flight and eliminates the concern expressed[2] regarding flow seeding effects on model heat transfer, flow chemistry, and possible particle damage to models. This paper very briefly presents some additional measured MHD interaction effects flow data obtained from the earlier study in order to extend the scope of useful information available to interested investigators and wind tunnel users.

Discussion

This study was conducted using a high-performance electrically driven reflected shock tunnel. The unique performance capability of this type of shock driving technique permitted the generation of high pressure–high temperature shock tunnel reservoir conditions yielding highly ionized unseeded air flows in the accelerator and nozzle with moderate values of chemical dissociation.[3,4] The over-all facility setup is illustrated in Fig. 1. The design and performance of the MHD accelerator were based upon a one-dimensional, steady, inviscid, compressible flow analysis.[5] A typical set of calculated performance curves for this experiment is given in Fig. 2.

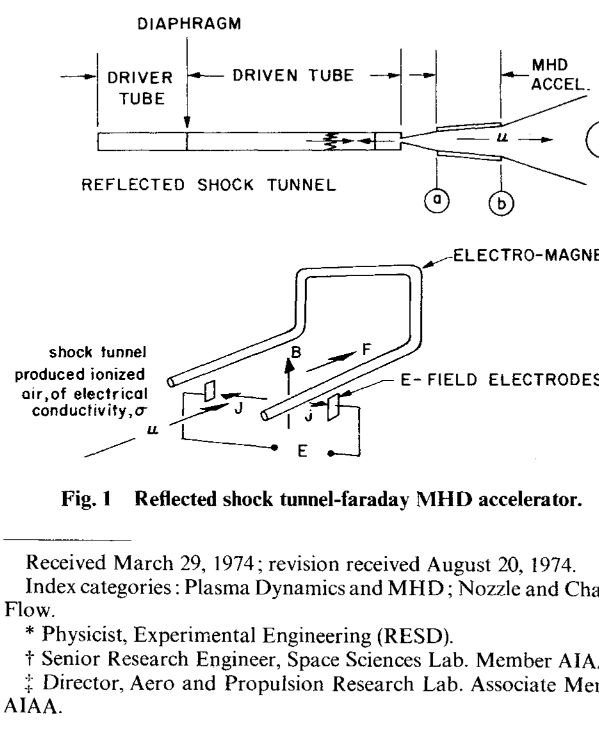

Fig. 1 Reflected shock tunnel-faraday MHD accelerator.

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